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The Comparative Study of Gray Model and Markov Model in Pavement Performance Prediction

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ABSTRACT
Pavement performance prediction is an essential component of pavement maintenance management system, which directly affects the choice of maintenance measures and funds. Firstly, this paper uses the Gray theoretical model to predict the status of certain highway pavement damaged. Secondly, the Grey theory and Markov prediction method are combined to forecast it. Finally, the comparison of the results between Grey theory and Markov prediction method analyzes the similarities and differences .The results show that Grey theoretical model is more suitable for recent forecasting, while combination method is suitable for longer-term forecasting.
Keywords -gray prediction model，Markov prediction model，pavement condition index

I. INTRODUCTION

There are several methods to predict pavement conditions such as deterministic model, probabilistic model, Grey theory and neural networks [1]. Currently, most scholars have no longer use a single model to predict the performance of the road, but take an integrated approach to predict it in the all-round consideration on the basis of the characteristics of each model, such as the Gray Markov model[2], Neural Network and Markov combined forecasting model[3],Gray neural network model[4].Because of the complexity, diversity and variability of the road system, how to find the most suitable method for the decay characteristics of each road performance will be the focus of our study and directly have an impact on road life and economic investment.

Grey system theory is a method that study the things What is uncertain. It is based on exploration about the less date of characteristic, behavior and potential regular to reveal the law of things in the context of less data, less information[5].The prediction model of GM (1,1) based on few data , referred to the Grey prediction.

Probability prediction model which is represented by Markov model considered the uncertainty of pavement performance prediction. It can better able to reflect the uncertainty of changes in various factors that result in pavement performance changes, which is more in line with the actual situation. Markov probability forecasting model as the starting point for a variant model, according to the latest survey data as a starting forecasting point to predict and improve the prediction accuracy, what’s more, the model can be updated with changes in road conditions [6].

II. TEXT

Some dates which is collected in Chongqing from 2001 to 2005 on behalf of expressway asphalt pavement condition index (PCI) as follows: 98.4,93.3,90.7,87.5,83.4. The condition of the roads damaged in the coming years is predicted.

1. Using GM (1,1) Model to predict

(1)Exploration about applicability of GM(1,1) Model

The original data sequence:

$X^{(0)}$ =（98.4 93.3 90.7 87.5 83.4）

Accumulation of the original data sequence once time:

$X^{(1)}$ =（98.4 191.7 282.4 369.9 453.3）

Smooth text:

$$\rho(k)=\frac{x^{(0)}(k)}{x^{(1)}(k-1)}, k=2,3,....n \quad (1)$$

When $\rho(k)$ is less than 0.5, to meet the conditions of quasi-smooth.

According to equation (1),the results as follows:

$\rho(2)=0.948, \rho(3)=0.473, \rho(4)=0.310,$

SO when K is greater than 3, to meet the conditions quasi-smooth.

Quasi-exponential discipline:

$$\sigma^{(1)}(k)=\frac{x^{(1)}(k)}{x^{(1)}(k-1)} \quad (2)$$

When $\sigma(k)$ is included in the interval of a and b , δ is equal to b minus a,and δ is less than 0.5, $X^{(1)}$ which establishes their GM (1,1) model is quasi-exponential.

According to equation (2),the results as follows:

$\sigma^{(1)}(3)=1.437, \sigma^{(1)}(4)=1.310, \sigma^{(1)}(5)=1.225$

When $k<3, \sigma^{(1)}(k)\in[1,1.5]$ and $\delta<0.5$, therefore, we can establish GM (1,1) model of $X^{(1)}$ sequence.

(2)Establishment of GM(1,1) Model

Generating close to the value of $X^{(1)}$sequence:

$Z^{(1)}$ =（145.05 237.05 326.15 411.6）

Using the least squares method to Solve responsive parameters:

$$B=\begin{bmatrix}-Z^{(1)}(2) & 1\\ -Z^{(1)}(3) & 1\\ \vdots & \vdots\\ -Z^{(1)}(n) & 1\end{bmatrix}=\begin{bmatrix}-145.05 & 1\\ -237.05 & 1\\ -326.15 & 1\\ -411.6 & 1\end{bmatrix}, Y=\begin{bmatrix}x^{(0)}(2)\\ x^{(0)}(3)\\ \vdots\\ x^{(0)}(n)\end{bmatrix}=\begin{bmatrix}93.3\\ 90.7\\ 87.5\\ 83.4\end{bmatrix}$$

$$\hat{a}=(B^TB)^{-1}B^TY=(a,b)^T=(0.0369457,99.068423)$$

Time response formula:

$$\hat{X}^{(1)}(k+1)=-2583.057e^{-0.0369457k}+2681.457 \quad (3)$$

According to equation (3),the results as follows:

Simulation value of $X^{(1)}$sequence:

$$\hat{X}^{(1)}=(98.4 \quad 192.09 \quad 282.38 \quad 369.40 \quad 453.26)$$

Simulation value of $X^{(0)}$sequence:

$$\hat{X}^{(0)}=(98.4 \quad 93.69 \quad 90.29 \quad 87.02 \quad 83.86)$$

(3)Test model accuracy

Accuracy of the model test results are shown in Table 1.

Table 1:Accuracy of the model test results

year	actual data	Analog data	Residuals ε(k)	Relative error Δk	mean square deviation C
2001	98.4	98.4	0	0	S1=5.091 S2=0.3904 C=S1/S2=0.0767<0.35 one Grade
2002	93.3	93.69	-0.39	4.18×10^{-3}	
2003	90.7	90.29	0.41	4.52×10^{-3}	
2004	87.5	87.02	0.48	5.49×10^{-3}	
2005	83.4	83.86	-0.46	5.52×10^{-3}	

Small error probable: $p=p\left\{|\varepsilon^{(0)}(i)-\bar{\varepsilon}^{(0)}|<0.06745S_1\right\}=1$

(4) PCI forecasting value according to GM(1,1)

Table 2: PCI forecasting value according to GM(1,1)

year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Measured value	98.4	93.3	90.7	87.5	83.4						
Predicted value	98.4	93.69	90.29	87.02	83.82	80.82	77.89	75.06	72.34	69.72	67.19
Grade	one	one	one	one	two	two	two	two	two	three	three

Table 2 shows that pavement conditions will drop to intermediate level when in 2010,it should take measures or medium-capital overhauling in advance.

2.Using Gray Markov model to predict

(1)Determining state distribution of pavement condition

According to Gray theoretical predictions, state distribution of pavement condition can be calculated by adopting attribute measure on a basis of measured value in 2005 and predictive value in 2006.

measure attribute of 2005:

P_0=（0.893　1　0.107　0　0）

measure attribute of 2006:

P_1=（0.721　1　0.279　0　0）

Normalized:P_0=（0.447　0.5　0.054　0　0），P_1=（0.361　0.5　0.140　0　0）

(2)Determining transition probability matrix as follow:

$$p_1 = p_0 \times p = p_0 \times \begin{bmatrix} p_{11} & p_{12} & & & \\ & p_{22} & p_{23} & & \\ & & p_{33} & p_{34} & \\ & & & p_{44} & p_{45} \\ & & & & 1 \end{bmatrix} \quad (4)$$

According to equation (4),the results as follows:

$$p = \begin{bmatrix} 0.807 & 0.193 & & & \\ & 0.828 & 0.172 & & \\ & & 0.998 & 0.002 & \\ & & & 0.937 & 0.063 \\ & & & & 1 \end{bmatrix}$$

(3)PCI forecasting value according to Gray Markov model is shown in Table3.

Table 3: PCI forecasting value according to Gray Markov model

year	Status distribution	PCI	Grade
2006	P_1=（0.361　0.5　0.140　0　0）	80.82	two
2007	P_2=（0.291　0.484　0.224　0.001　0）	78.28	two
2008	P_3=（0.235　0.456　0.307　0.002　0）	75.79	two
2009	P_4=（0.189　0.423　0.385　0.003　0）	73.45	two
2010	P_5=（0.152　0.386　0.457　0.005　0）	71.29	two
2011	P_6=（0.123　0.348　0.522　0.007　0）	69.34	three

TABLE 3 shows that pavement conditions will drop to intermediate level about in 2011,it should take measures or medium-capital overhauling in advance.

3.Comparison of predicted results

TABLE 2 and TABLE 3 indicate that the prediction results which are based on model of GM (1,1) and Gray Markov are consistent with damaged pavement condition decay law. The difference is the rate of decay. Grey theory predictions decay rates is faster than the Gray Markov.

There are two main reasons, one is that the original forecasting data is different. Gray model which is based on the original survey data is relatively long, Grey Markov forecasting is closer to the future development of pavement on the basis of theoretical predictions, therefore, its decay results are more in line with the development of road conditions. Another reason is the characteristics of the model itself. Gray theory prediction model is more suitable for short-term prediction when the original data is less and the accuracy of gray theory prediction model is limited. The combination of Gray theory together with the Markov model forecast road conditions can not only take full account of the development of the uncertainty, but also make the model for longer-term predictions with the development of the road constantly updating.

III. CONCLUSIONS

Due to the complexity of factors that affect pavement performance, a single prediction method does not receive good results. Therefore, a combination of a variety of methods will be used in more pavement performance prediction. The probabilistic forecasting model as a variable starting point prediction model can better meet the actual situation, gray theory prediction model can overcome road information "black and white" nature. Examples show that the combination forecasting of the two methods which is more in line with the decay of pavement properties can give full play to their respective advantages and make the prediction accuracy higher. Grey theoretical model is more suitable for recent forecasting, while combination method is suitable for longer-term forecasting.

GM (1,1) model is established on the basis of the original data, the data model cannot be updated, it will result prediction accuracy poor in longer years . Therefore, future studies can be constantly updated by forecasting model based on the new data by consecutive surveys. The State of Markov transition matrix model is static, but the difference exists between the state transition to another state, ignoring this difference is bound to increase the prediction error. In this regard, the model can be used to improve prediction accuracy, especially, the state transition matrix becomes a dynamic matrix, constantly updated transition probability matrix.

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